Prediction of Tank Fuel Sloshing during Jettison

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ABSTRACT

Separation Clearances for tanks provide in general the most limiting conditions for the emergency jettison envelope of a fighter aircraft.

Tanks are almost wide body but light weighted stores with poor aerodynamic characteristics as far as stability is concerned. Especially when partially filled, the physical characteristics of the residual fuel become an additional driver for the separation behavior. There, its physical properties become a function of the aircraft flight attitude, depending on angle of attack, flight path angle and bank angle. During the trajectory itself these properties rapidly change in dependence of the tank motion. Therefore store separation analysis of partially filled fuel tanks always represents a delicate problem.

The target of this paper is to highlight a scheme that allows the representation of fuel sloshing effects in a fuel tank during separation. The concept of this scheme consists of two steps, a steady state calculation using finite elements for the fuel volume, and a dynamic analysis using a state of the art separation code used at EADS Deutschland (SSP) .It will be shown that the center of gravity and the moments of inertia rapidly change in dependence of the tank motion. In order to represent these effects, analytical results for a sloshing liquid in cylindrical volumes are used to predict the effective sloshing mass, depending on the rotation rate of the body and the body geometry. During a jettison process the relaxation time for the high accelerations and rates are taken into account as deviation from their pseudo-steady position. Fitting this scheme into the separation code, trajectories with / without sloshing fuel mass effects of an external tank are presented and the differences are lined out.

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0.0 ABBREVIATIONS & SYMBOLS

a : shape parameter paraboloid [1/m]

b : axis parameter paraboloid [m]

c : axis parameter plane [m]

E_{kin} : kinetic energy [kJ]

E_{pot} : potential energy [kJ]

I,j : loop indices [-]

I_{xx} / I_{xx} / I_{xx} : moments of Inertia around x- / y- / z axis [kg*m^2]

I_{eff} : effective moment of Inertia [kg*m^2]

I_{solid} : moment of Inertia of a rigid body [kg*m^2]

L : $L = E_{kin} + E_{pot}$ (Lagrange's formula)

$L_{i,\,eff}$: quasi-particle effective angular momentum [kg*m^2 / s]

L_{eff} : overall effective angular momentum [kg*m^2 / s]

L_{solid} : angular momentum [kg*m^2 / s], rigid body

m : slope [-]

m, n : Lennard Jones potential parameter [-]

mass : quasi-particle mass [kg]

nz : load factor [*g0, Earth gravity]

N_{circ} : overall number of circular segment elements [-]

N_x : overall number of axial segment elements [-]

NT : number of quasi-particles [-]

Q_{qi} : exterior forces [N]

q_i : generalized coordinates (Lagrange's formula)

R : cylinder radius [m]

r_i : radius quasi-particle i from (0/0/0) [m]

r_{ij} : radius quasi-particle i to quasi-particle j [m]

U_{ij} : Lennard-Jones potential function [kJ]

v_i : velocity quasi-particle i [m / s]

x , y , z : Cartesian coordinates [m]

x_{cg} , y_{cg}, z_{cg} : center of gravity [m]

x_i , y_i , z_i : quasi-particle coordinates [m]

y_0 , y_1 , z_0, z_1 : limits of integration [m]

α : potential parameter: $\alpha = f(m, n)$

ε : equilibrium rest potential [kJ]

γ : flight path angle [deg]

Ω : angular velocity [rad / s], x-axis

φ : roll angle [deg]

Θ : inclination angle [deg]

σ : quasi-particle diameter [m]

operators:

$\tilde{I}$: tensor of moments of inertia

$\vec{a} \times \vec{b}$: vector product of *a* and *b*

1.0 INTRODUCTION

The behaviour of liquids in aerospace vehicles like airplanes, rockets or satellites has always been a delicate problem. The fuel provides a significant contribution to the overall mass of these vehicles, and therefore the knowledge of the mass properties of the fuel is extremely important for performance and safety considerations. In the domain of military aircraft, high accelerations occur during flight, where the fuel is forced to slosh within the tanks, especially in partially filled compartments. In order to accurately predict the trajectory of a separating fuel tank in vicinity of an aircraft, it is of paramount importance to represent the correct physical properties of the tank in combination with the ejection and the aerodynamic forces inclusively all interference effects. Thereby the liquid motion becomes the driving parameter for the changing physical properties of the tanks.

This paper deals therefore with an approach that allows a very fast determination and representation of liquid sloshing effects and terms on the jettison of a partially filled external fuel tank.

2.0 LIQUIDS

In contrast to a solid body, liquid does not keep its shape when it is in motion. The intermolecular forces are weaker than in the crystal grids of a solid, so there is no global order that preserves coherence. In contrast to a gas where nearly no coherence at all exists between the molecules, there are regions where a certain number of molecules are closely coherent. These regions can easily break up and re-connect, and therefore the liquid may take every shape the boundary prescribes. The strength of the intermolecular forces varies from liquid to liquid, dominating a property that is a driving factor for all dynamical analysis: viscosity. Viscosity strongly depends on the liquid temperature and pressure, whereas in a wide range of pressure and temperature, a liquid can be idealized as incompressible. Thus, the volume can be estimated as constant, whereas the shape changes [1].

3.0 SIMULATION APPROACHES

The modelling of fuel sloshing effects within a tank is a large area for scientific research. Several approaches have been done, analytically and in experiments, to predict the effective sloshing mass and the effective moments of inertia of the liquid, [2], [3], [4], [6]. With a liquid consisting of billions of particles (ions, molecules), molecular dynamics offers in theory the best approach to sloshing effects. But with the extreme big amount of particles and the complexity of solving the governing, partly statistical equations, other approaches have become more and more popular as the computer power has rise as well.

Continuum fluid mechanics, namely Euler- and Navier-Stokes codes, have proved to provide accurate models for gas dynamics and aerodynamic simulation. But in terms of liquids, especially rapidly moving liquids, the codes are extremely time consuming, due to the extreme small time steps required for proper integration. Analytical approaches have been done as well, transforming the physical problem into a mathematical one. These approaches are mostly only valid for idealized liquids (no friction, incompressible, vortex-free, a.s.o.), so they are often limited to special problems [5].

A third way can be considered which deals with macroscopic particle clusters, containing billions of real molecules. The properties of the real molecules are transformed to the macroscopic particles, having dimensions of some 10E-02 meters (instead of 10E-15 meters for real molecules). Some results from this particle approach are presented in figures 7 to 9 in order to demonstrate their relevance for a sloshing motion. Although this particle approach provides very good results for slosh predictions, see figure 8 and 9, it still consumes more CPU time than accepted for an effective trajectory analysis. Therefore a special concept based on elements derived from quasi particle cloud models is used within this paper in order to compute trajectories in reasonable computing times and much faster than achievable with the classical approaches.

4.0 STEADY STATE MODEL

To predict the centre of gravity shift due to the motion of the liquid within the tank, an analytical model has been considered for a typical three-compartment tank. Within the consideration, the overall tank volume is divided up into three compartments, one central, main, cylindrical compartment and two parabolic ones at the extremities (figure 1). The compartments are enumerated as followed:

front compartment:	compartment 1
middle compartment:	compartment 2
rear compartment:	compartment 3

Each of these compartments can be individually and arbitrarily filled. In our present example, each compartment is assumed to be filled at the same fuel level.

Inclining the tank as shown in figure 2 the volume occupied by the liquid becomes a function of the free surface intersection of the liquid with the solid walls and the inclination angle. Separating the fuel volume into finite elements, the center of gravity and the moments of inertia for each element are computed and summed up to overall values for the partially filled tank at this inclination. In such a way, steady state mass characteristics of the fuel can be obtained at several different inclinations. With the tank partially filled, the fuel will have a free surface that is always orientated vs. Earth gravity field. Supposing the liquid in equilibrium at every time step, the center of gravity changes with inclination of the tank, due to the change of the liquid shape, see figure 3.

The three compartments have different geometry and can be filled individually. Thus, each compartment has to be analysed isolated from the others and the resulting center of gravity for the overall liquid volume is obtained by superposition. With the tank and the liquid moving in 6 Degrees of Freedom after jettison, sloshing effects in three dimensions have to be considered.

Bearing in mind the assumption of a smooth, free surface of the liquid, always oriented vs. Earth gravity field, the orientation of the tank walls is different from the orientation of the free liquid surface, see figure 2. Therefore a coordinate transformation is necessary to bring the two mass systems into congruency so that the store trajectory can be computed for the overall system consisting of tank walls and liquid.

4.1 Finite Elements Approach

The fuel takes the volume between the free surface and the tank walls. In mathematics, the volume is the integral between a plane (representing the free surface) and a boundary function (representing the tank shape).

For the two parabolic compartments, the overall volume occupied by fuel is given by the volume integral

$$Vol_{Paraboloid} = \int_{y0}^{y1}\int_{x0}^{z1} x(y,z)\cdot dy\cdot dz \quad \text{(axis orientation as shown in figures 1 and 2)}$$

In Cartesian coordinates, x can be expressed as a function of y and z

$$x = -a\cdot(y^2 + z^2) + b$$

with b denoting the axis parameter in x-direction and a being a coefficient for the shape of the paraboloid. With R representing the radius of the cylindrical compartment of the tank, a is a function of b and R:

$$a = \frac{b}{R^2}$$

In the present example, the values for R, a and b are

$$R = 0.37m, \qquad a = 12.8, \qquad b = 2.05$$

A plane, orientated towards the tank walls by an inclination angle Θ, represents the free liquid surface. In Cartesian coordinates, this plane is given by

$$x = m\cdot y + c$$

Axis orientation is the same as for the paraboloid. m denotes the plane inclination, c the axis parameter with the x-axis.

m is given by $m = \tan(90° - \Theta)$,

c depends on the fill level of the compartment.

The plane boundary intersection in z-direction is

$$-a\cdot\left(y^2 + z^2\right) + b = m\cdot y + c \quad \Leftrightarrow z = \pm\sqrt{-y^2 - \frac{m}{a}\cdot y + \frac{b-c}{a}}$$

The intersection in x-direction is

$$-y^2 - \frac{m}{a}\cdot y + \frac{b-c}{a} = 0 \quad \Leftrightarrow y_{1,2} = -\frac{m}{2\cdot a} \pm \sqrt{\left(\frac{m}{2\cdot a}\right)^2 + \frac{b-c}{a}}$$

The liquid volume for the parabolic compartments then results in

$$Vol(y,z)=\int_{y_1=-\frac{m}{2\cdot a}-\sqrt{\frac{m^2}{4\cdot a^2}+\frac{b-c}{a}}}^{y_2=-\frac{m}{2\cdot a}+\sqrt{\frac{m^2}{4\cdot a^2}+\frac{b-c}{a}}}\int_{z_1=-\sqrt{-y^2-\frac{m}{a}\cdot y+\frac{b-c}{a}}}^{z_2=+\sqrt{-y^2-\frac{m}{a}\cdot y+\frac{b-c}{a}}}\left(Paraboloid(y,z)-Plane(y,z)\right)\cdot dy\cdot dz$$

An analogous formula arises for the cylindrical compartment, replacing the paraboloid by the boundary function for a cylinder. This volume integral cannot be evaluated easily but has to be valid for the overall definition sector for tan(Θ) and the square functions, which is, obviously, not possible. An analogous formula arises when deriving the moments of inertia for the liquid volume.

Therefore, an approximation is introduced which allows an accurate and fast computation of the center of gravity and the moments of inertia: The volume is separated in finite elements (see figure2).

4.1.1. Discretisation

Starting from the volume integral, the liquid volume is discretized with finite elements. To simplify the computation of the moments of inertia, finite elements of rectangular geometry have been chosen. By this, the finite volume can be evaluated easily and the moments of inertia are well defined. Therefore, bars are used to represent the liquid volume in the compartments. The volume integral is split of in two plane integrals, one with respect to the *x*-*y* plane and the other one with respect to the *y*-*z* plane (for the axis orientation see figures 1 and 2). As the tank body is axially symmetrical, every intersection of the liquid plane and the boundary in the *y*-*z* plane for a given *x* is a circle segment. The area of this segment is computed using the trapezoidal rule with N_{circ} steps. Discretizing the liquid volume in *x*-direction in the same way using N_x steps, and summing up the discrete circle segments for all *x* gives the overall volume of the liquid. For the computation, the following discretization has been used:

$N_{circ} = 100$, $N_x = 100$, $N_{compartment} = 10.000$ elements

Depending on the desired fill level the volume of the liquid is adjusted by variation of the axis parameter of the free surface plane, *c*. For the computation of the volume, first a start axis parameter has to be chosen and the volume within this intersection space is computed. The parameter is varied if the volume computed does not fit to the desired volume. Thus, the final axis parameter is determined by an iteration loop. With the parameter fixed, the center of gravity of the liquid can be computed using the center-of-mass law. In a second step, the moments of inertia referred to this center of gravity are determined, taking into account the Steiner parts of the finite elements. This scheme is repeated for every compartment, the overall center of gravity for the liquid mass in the tank is obtained by superposition. The resulting center of gravity for various inclination angles Θ is shown in figure 3. Obviously the center of gravity shift in axial direction is in the order of 0.5 m for a half-filled tank.

4.1.2 Polynomial Interpolation

With the inclination angle Θ changing rapidly during jettison, the iteration loop to compute the liquid surface axis intersection has to be run for every time step in the integration of the trajectory. This would be very time-consuming and ineffective. Therefore, the intention is to provide the mass properties of the liquid in smooth functions that can be evaluated much faster than an iteration loop. In order to determine these functions, the mass properties (center of gravity, moments of inertia) for several inclination angles Θ between –90 deg and +90 deg have been computed in dependence of the individual fill level for every compartment. The results from these computations for the rear compartment and the fill levels 1/3 full, 2/3 full, 3/3 full are shown in figures 4, 5, 6.

A functional dependence between the inclination angle Θ, the fill level and the mass properties of the liquid can be found by polynomial interpolation: A 6^{th} order polynomial is used to approach the dependence accurately. With polynomial functions being smooth and well defined, the center of gravity and moments of inertia can now be provided for every inclination angle Θ within the definition interval, see figures 4, 5, 6. In addition to that, even fill levels that are not conform to 1/3, 2/3 and 3/3 are provided by interpolation.

5. DYNAMICAL ANALYSIS

5.1 Effective Moments of Inertia

The moments of inertia computed with the finite element approach as mentioned above are only valid for a solid liquid: The liquid is supposed to be in equilibrium, with a plane free surface at every time step of the integration. The liquid is supposed to rotate at the same rotation rate as the tank body.

Real liquids, in contrast, are not in equilibrium when in motion. A lot of the kinetic energy is transformed into internal energy. In the molecular domain, this energy is transformed into oscillations of the particles. In regions where these oscillations are strong enough intermolecular connections break up and reunify.

The driving parameter for the change of the liquid shape is the moving boundary: The liquid takes the shape the boundary offers. This is true e.g. for a tank inclination where the section walls prevent the liquid from sloshing into the next compartment. The section walls interact with the fuel, pushing it into a new position. The same effect occurs for liquid motion forced by stringers or other elements of the tank structure. The moments of inertia computed with the finite elements approach are only valid for this case of a change of shape forced by the boundary geometry.

If there is no geometry change driving the liquid motion, e.g. in the case of roll excitation, supposed there are no buffers that keep the liquid rotating, the only driving parameter for liquid motion is the contact line between the liquid and the wall. With the friction forces between the liquid and the wall small but not equal zero and the cinematic viscosity of the fuel sufficiently small and not equal zero as well, only the fuel mass close to the wall is forced to rotate with the wall velocity, whereas the fuel mass away from the walls does not rotate at the same level (“differential rotation”).

Therefore, bearing in mind a trajectory with quick rotations, the effective moments of inertia of fuel with respect to the tank roll axis are smaller than predicted by the finite element approach. The effective moments of inertia depend mainly on the fill level, the acceleration rate and the geometry of the tank.

5.2 Quasi-Particle Approach

In order to complete the finite element approach by the effective moments of inertia Ixx of a liquid, the particle approach is now used. For this purpose, a sufficient number of particles have been chosen that represent the fuel in a partially filled tank. The equation of motion of the particles is given by Lagrange’s formula

$$\left(\frac{\partial L}{\partial \dot{q}_i}\right)^{\bullet} - \left(\frac{\partial L}{\partial q_i}\right) = Q_{q_i} \quad q_i = x_i, y_i, z_i \ ; \ i = 1,2,\ldots,NT \quad \text{for} \quad i_T = 1,\ldots,N_T$$

with $L = E_{kin} + E_{pot}$; $E_{kin} = \frac{1}{2} \cdot mass \cdot \dot{q}^2$; $E_{pot} = \sum_{\substack{j=1 \\ j \neq i}}^{NT} U_{ij}$ and the generalised coordinates $q_i = x_i, y_i, z_i$

The potential energy is given by a Lennard-Jones potential

$$U_{ij} = \alpha \cdot \varepsilon \cdot \left(\left[\frac{\sigma}{r_{ij}}\right]^m - \left[\frac{\sigma}{r_{ij}}\right]^n \right)$$

where *m* denotes the exponent of the repulsing term and *n* the exponent of the attractive term.

Solving Lagrange's formula with the kinetic and potential energy mentioned above, the equation of motion results in

$$mass \cdot \ddot{q}_i - \sum_{\substack{j=1 \\ j \neq i}}^{N} \alpha \cdot \varepsilon \cdot \sigma^n \cdot n \cdot \left(\left(\sigma^{m-n} \cdot \frac{m}{n} \cdot \frac{1}{r_{ij}^{\ m+2}} - \frac{1}{r_{ij}^{\ n+2}} \right) \cdot \left(q_i - q_j\right) \right) = Q_{q_i}$$

with the forces Q_{qi} representing gravity, tank acceleration, a.s.o.

Solving this equation for every particle and integrating the equations with a 6th order Runge-Kutta-Fehlberg formula (table 2), the dynamics of the fuel sloshing can be predicted very accurately.

The following parameters have been used for the computations (each compartment filled at 1/3):

NT (compartment 1) = 2250, NT (compartment 2) = 4500, NT (compartment 3) = 2500

σ= 0.03 m, mass = 0.03 kg, ε = 0.0001 kJ, m = 12, n = 6, α = 4

The geometry for the particle approach is shown in figure 7. The boundary of the three compartments is represented by quasi-particles coloured in red, the fuel particles coloured in blue.

Summing up the rotational energy of the particles with respect to the tank axis at every time step and building an average momentum for the particle clusters due to a forced exterior oscillation yields to the effective moments of inertia: The angular momentum of every particle is

$$\vec{L}_{i,eff} = mass_i \cdot (\vec{r}_i \times \vec{v}_i) \qquad , \qquad \vec{L}_{eff} = \sum_{i=1}^{NT} \vec{L}_{i,eff} = \sum_{i=1}^{NT} mass_i \cdot (\vec{r}_i \times \vec{v}_i)$$

The angular momentum for a solid fuel is

$$\vec{L}_{solid} = \tilde{I}_{solid} \cdot \vec{\Omega}_{\tan k} = \sum_{i=1}^{NT} mass_i \cdot r_i^{\,2} \cdot \vec{\Omega}_{\tan k}$$

with $\vec{L}_{eff} = \tilde{I}_{eff} \cdot \vec{\Omega}_{\tan k}$

the ratio $\dfrac{\left\|\tilde{I}_{eff}\right\|}{\left\|\tilde{I}_{solid}\right\|}$ becomes $\dfrac{\left\|\tilde{I}_{eff}\right\|}{\left\|\tilde{I}_{solid}\right\|} = \dfrac{\left\|\vec{L}_{eff}\right\|}{\left\|\vec{L}_{solid}\right\|}$.

The results have been compared to some analytical investigations that have been carried out to predict the effective moment of inertia of a liquid. One example for an effective moment of inertia is given in figures 8 and 9. The effective moment of inertia Ixx, averaged by a power method, is reduced to about 40 percent of the value for a solid body. Using this approach for several acceleration rates, the effective moments of inertia with respect to the x-axis are obtained in dependence of angular rate and fill level. This functional dependence is implemented into the finite element scheme, reducing the roll moments of inertia from solid liquid to a real liquid.

5.3 Implementation in the Store Separation Program (SSP)

The core of this investigation consists of the *EADS* Store-Separation-Program (SSP), a 6 DOF simulation program that computes the motion of jettisoned or launched stores and missiles in any type of interference flow around the carrier aircraft. This approach is based on different mathematical modelling strategies which are utilising computed sectional loads and aircraft flowfields, as well as measured installed and on time accurately computed End-Of-Stroke loads in order to represent the aerodynamic effects. The theoretical background is based on Euler solutions successfully in use at *EADS* Deutschland for this special purpose since more than 15 years; see [7], [8]. The slosh subroutine is implemented in the SSP main program using the actual center of gravity and flight path angles of the tank motion at every time step. For these values, the subroutine computes the new center of gravity and moments of inertia for the liquid mass and superposes these results with the tank boundary. The resulting mass properties are referred to the new center of gravity. Therefore the aerodynamic loads and velocities have to be transformed to the new center of gravity as well. After this transformation, the new position and orientation of the tank are computed within the next time step, the new values are used again as input for the slosh routine. This always relates the aerodynamic forces to the actual center of gravity of the resulting system of tank and liquid. For the computations, the tank properties in table 1 have been used.

6 RESULTS & CONCLUSION

Some trajectories for a partially filled three-compartment tank are shown in figures 10 to 15. For identical flight conditions, tank jettison trajectories for straight level flight, climb and dive are presented. In the present examples, the tank compartments are equally filled at 1/3 fuel level. The trajectories have been computed either with the fuel modelled as a solid or as sloshing fuel. The difference between the trajectories with / without fuel sloshing is obvious: The dynamical modelling of the fuel leads to a stronger nose down pitch compared to the solid model due to the sloshing fuel mass. With the initial nose down pitch momentum due to the Ejection Release Unit, the fuel in the three compartments is sloshing towards the tank nose. By this, the overall center of gravity also shifts towards the tank nose. With the reference point for the momentum computation and for the moments of inertia identical with the center of gravity, the tank becomes more stable compared to a fixed center of gravity. This nose down effect is clearly visible especially for the climb trajectory where the tank reaches quickly an upright position. The distance between the aircraft and the tank is growing faster compared to the prediction with solid fuel, providing a safer jettison attitude. In all three flight cases, the lateral movement of the tank with sloshing fuel is nearly reduced to zero compared to the solid approach. Bearing in mind the poor aerodynamic properties of an external fuel tank, the fuel sloshing reduces the aerodynamic instability to a certain degree, strongly dependent on the fill level.

In this analysis, the fuel has been regarded as a uniform mass always connected to the tank walls. In a next step, the simulation will be improved by a fuel model that takes into account separated fuel mass. With the tank accelerated very fast by the Ejection Release Unit, the fuel will separate from the walls and impact on the tank top. By this shock, the tank acceleration gets a peak, changing the further trajectory.

The quasi-particle model offers a good approach for separating fuel masses. The forces by the shock at the wall can be computed, too. Consequently, in a further study, the shock effects will be analysed in detail, defining parameters for the shock intensity that will be implemented in the slosh routine. Then, the slosh model will be almost complete to simulate all relevant fuel sloshing effects accurately.

7 FIGURES

Empty Tank	
Empty mass	125.0 [kg]
Volume	1.62 [m^3]
Xcg / Ycg / Zcg	2.90 / -0.001 / 0.04 [m]
Inertia Ixx / Iyy / Izz	8.9 / 195.0 / 195.0 [kg * m^2]

Fuel	
Fuel mass (total)	ca. 1180 kg
Rest Fuel mass	ca. 11.7 kg

Empty Tank + Fuel	
overall mass	ca. 1300 [kg]
Xcg / Ycg / Zcg	2.70 / 0.0 / 0.001 [m]
Inertia Ixx / Iyy / Izz (solid)	64.0 / 1890.0/1890.0 [kg * m^2]

Realistic Flight Conditions	
Tank empty with Rest Fuel:	135.0 [kg]
Tank full	1290 [kg]

Table 1: Mass properties of the tank / fuel system for the present analysis

1.1 $t_0, x_0, 0 \to x$

1.2 $t_1 = t_0 + \alpha_1 \cdot h$, $x_1 = x_0 + \dot{x}_0 \cdot \alpha_1 \cdot h + h^2 \cdot \gamma_{10} \cdot f_0$

1.3 $t_2 = t_0 + \alpha_2 \cdot h$, $x_2 = x_0 + \dot{x}_0 \cdot \alpha_2 \cdot h + h^2 \cdot \sum_{\lambda=0}^{1} \gamma_{2\lambda} \cdot f_\lambda$

1.4 $t_3 = t_0 + \alpha_3 \cdot h$, $x_3 = x_0 + \dot{x}_0 \cdot \alpha_3 \cdot h + h^2 \cdot \sum_{\lambda=0}^{2} \gamma_{3\lambda} \cdot f_\lambda$

1.5 $t_4 = t_0 + \alpha_4 \cdot h$, $x_4 = x_0 + \dot{x}_0 \cdot \alpha_4 \cdot h + h^2 \cdot \sum_{\lambda=0}^{3} \gamma_{4\lambda} \cdot f_\lambda$

1.6 $t_5 = t_0 + \alpha_5 \cdot h$, $x_5 = x_0 + \dot{x}_0 \cdot \alpha_5 \cdot h + h^2 \cdot \sum_{\lambda=0}^{4} \gamma_{5\lambda} \cdot f_\lambda$

1.7 $t_6 = t_0 + \alpha_6 \cdot h$, $x_6 = x_0 + \dot{x}_0 \cdot \alpha_6 \cdot h + h^2 \cdot \sum_{\lambda=0}^{5} \gamma_{6\lambda} \cdot f_\lambda$

2 $f_k = f(t_k, x_k)$ $\quad k+1 \to k$

k = 6

3 Truncation Error: $TE_{\dot{x}} = \frac{9}{280} \cdot (f_5 - f_6) \cdot h$

4 Not Correct: Take double or half stepsize , $1 \to k$

correct

5 Final Values: $t = t_6$, $x = x_6$, $\dot{x} = \dot{x}_0 + h \cdot \sum_{k=0}^{5} \dot{c}_k \cdot f_k$

6 Move: $t \to t_0$, $x \to x_0$, $f_6 \to f_0$, $1 \to k$

Table 2: 6th order Runge-Kutta-Fehlberg scheme, used for the particle approach

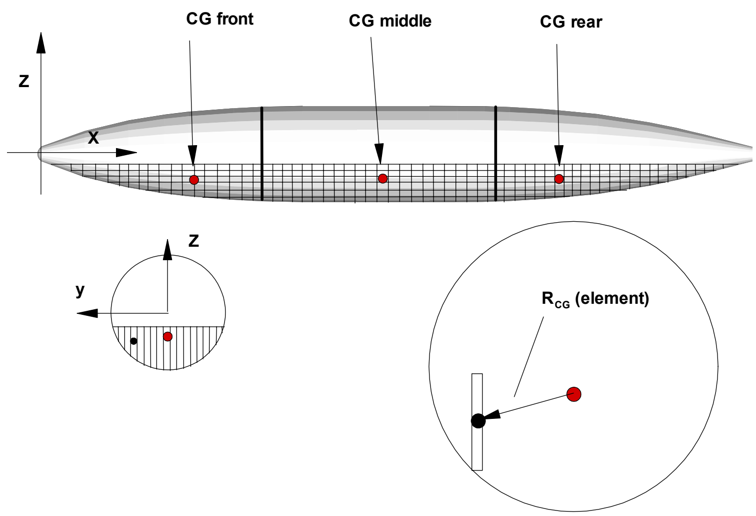


Figure 1: Tank partially and arbitrarilly filled

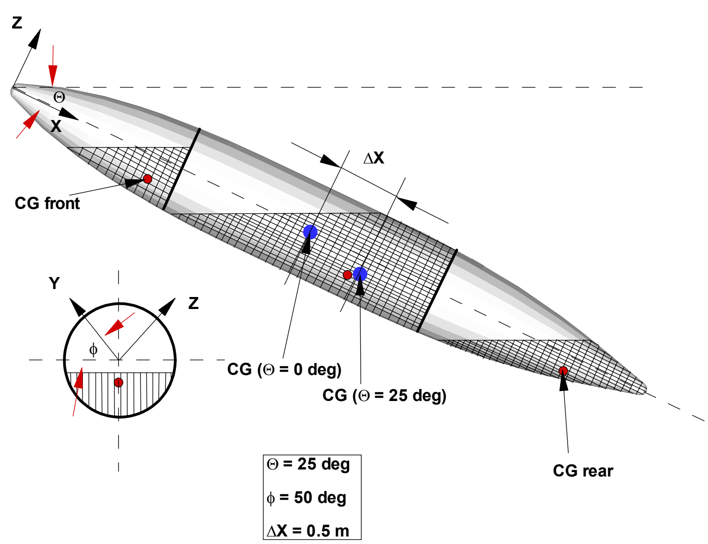


Figure 2: Partially filled tank inclined

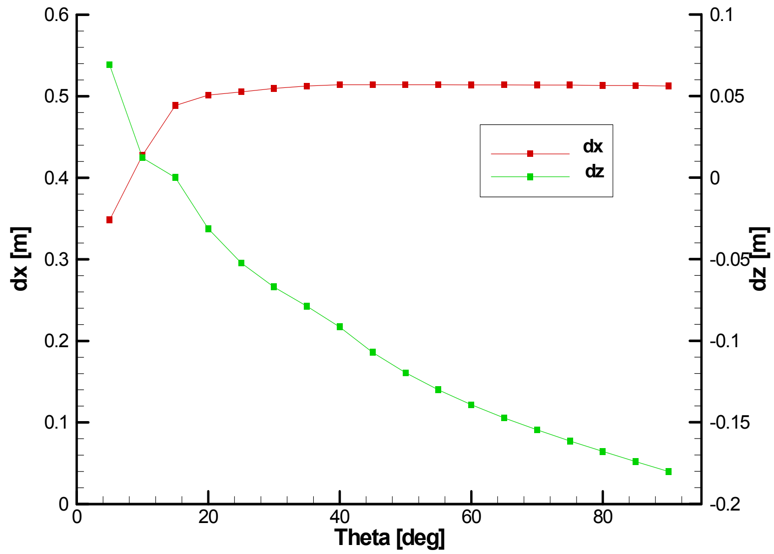


Figure 3: Center of gravity shift = f (inclination), each compartment filled at 1/2

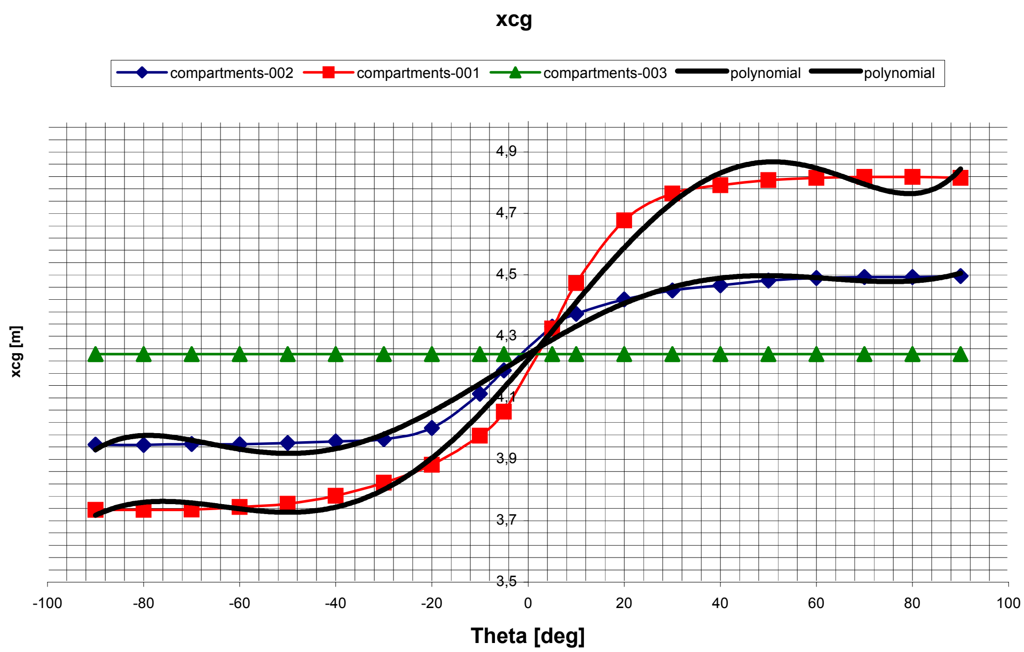


Figure 4: Axial center of gravity shift = f (inclination, fill level), compartment 3

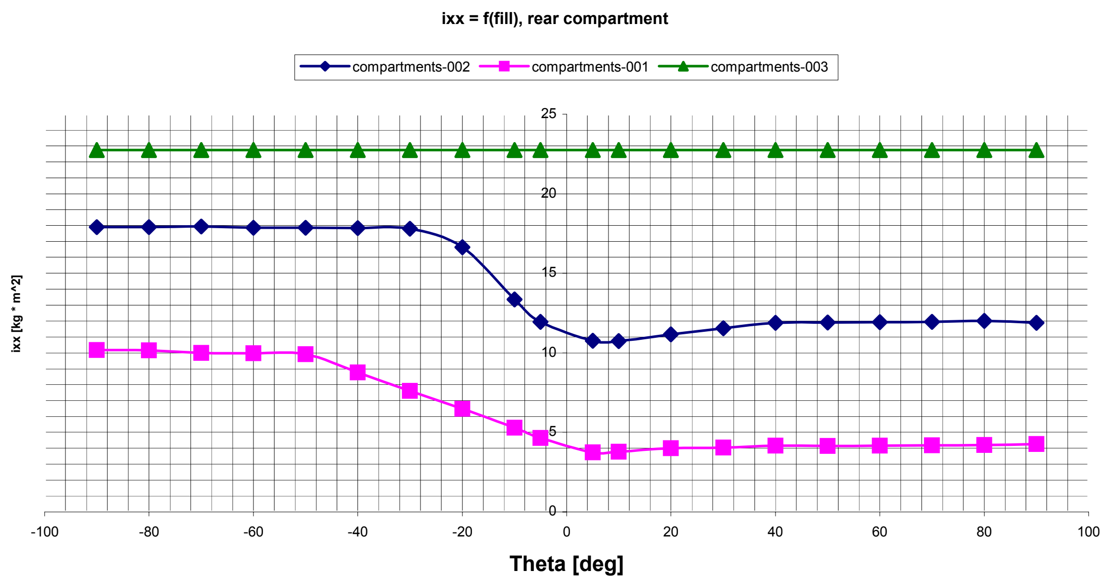


Figure 5-: Ixx_solid = f (inclination, fill level), compartment 3

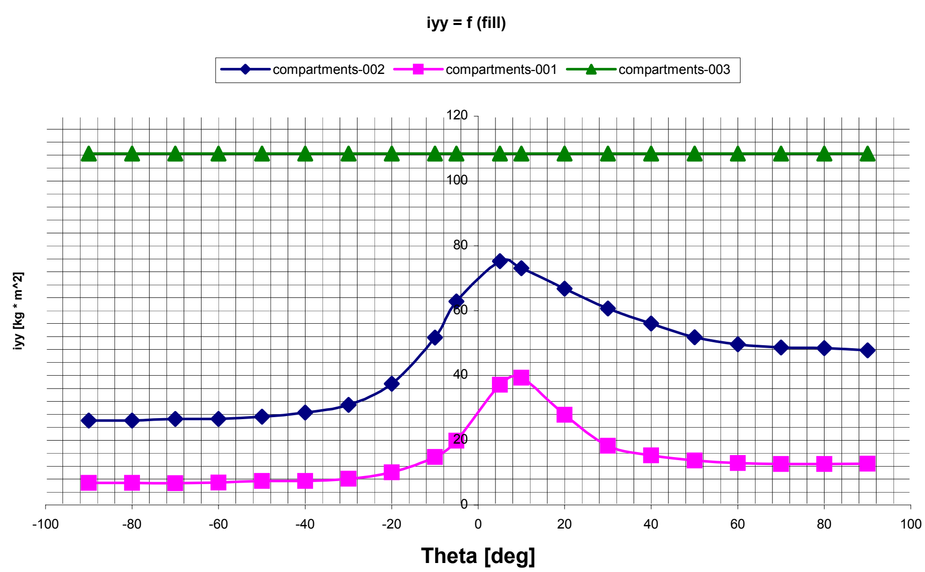
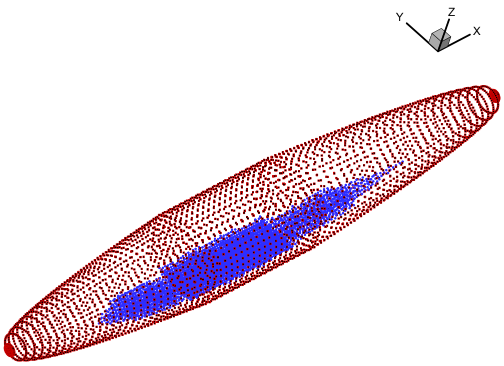
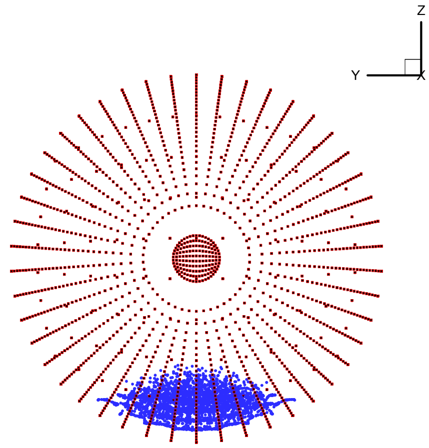


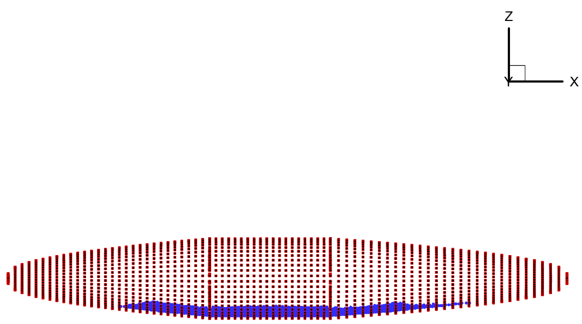
Figure 6: Iyy_solid = f(inclination, fill level), compartment 3



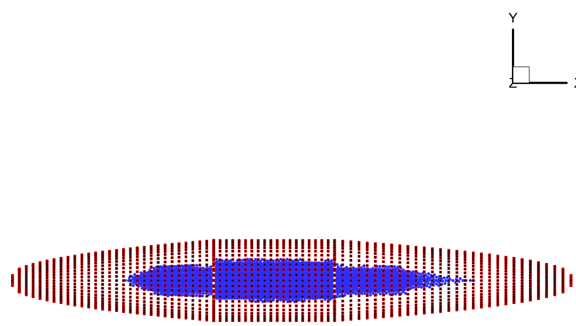
Quasi-Particle 3-compartment tank model, 3D view



Quasi-Particle 3-compartment tank model, front view



Quasi-Particle 3-compartment tank model, side view



Quasi-Particle 3-compartment tank model, top view

Figure 7: Quasi-Particle modelling of 3-compartment tank

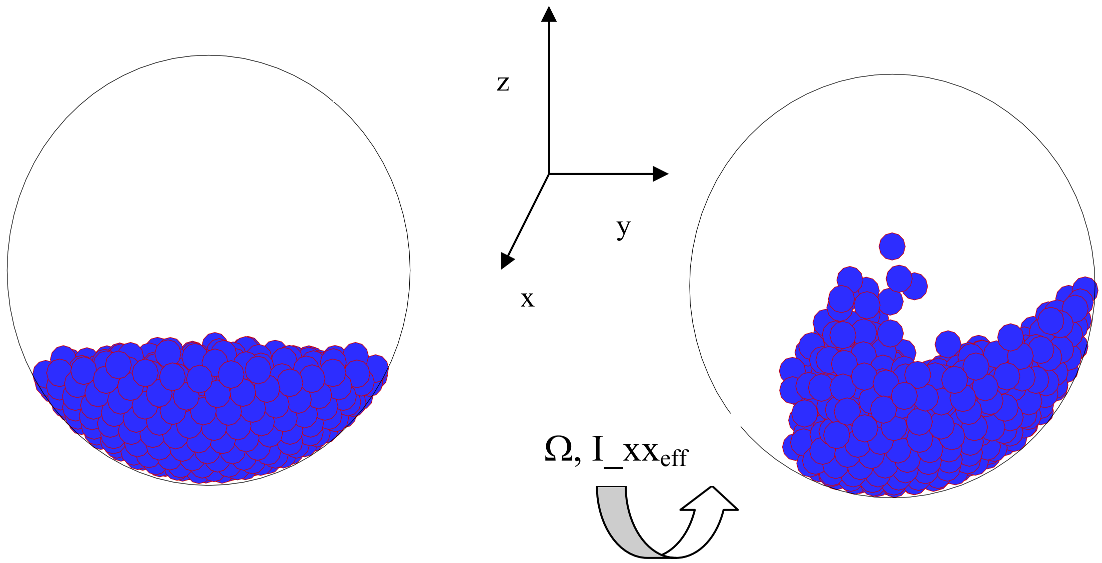


Figure 8: Partially filled tank cross section, in equilibrium (left) and sloshing (right)

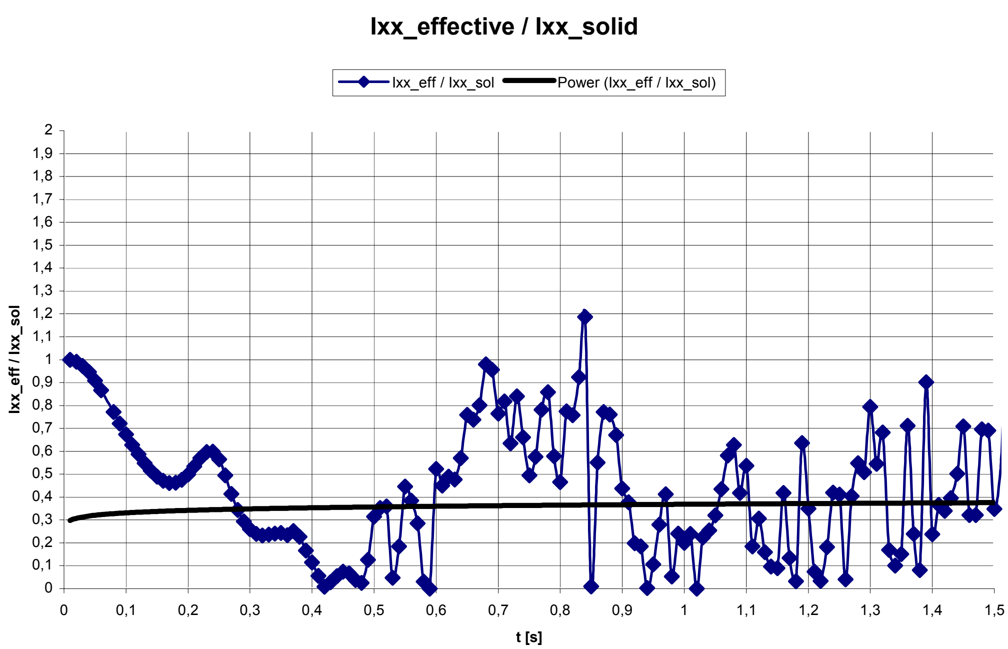


Figure 9: Ixx_eff / Ixx_solid = f (t), Ω = 2.*Pi [rad/s], compartment 1, filled at 1/3

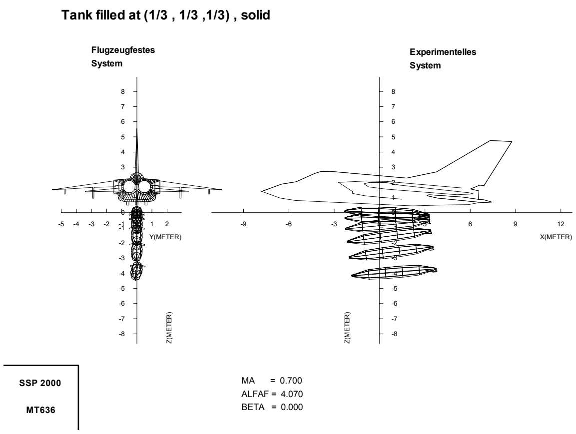


Figure 10: Trajectory without sloshing, partially filled tank, nz = 1g, γ = 0 deg

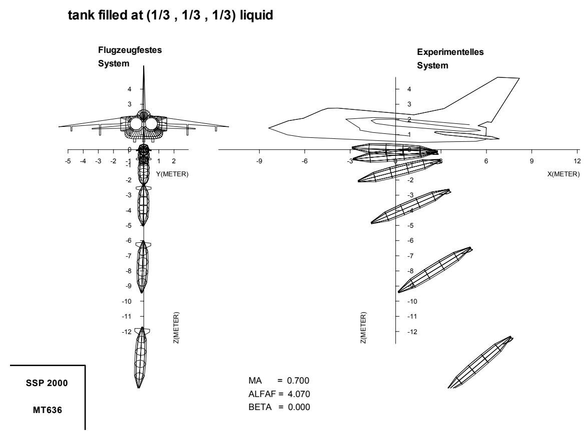


Figure 11: Trajectory with sloshing, partially filled tank, nz = 1g, γ = 0 deg

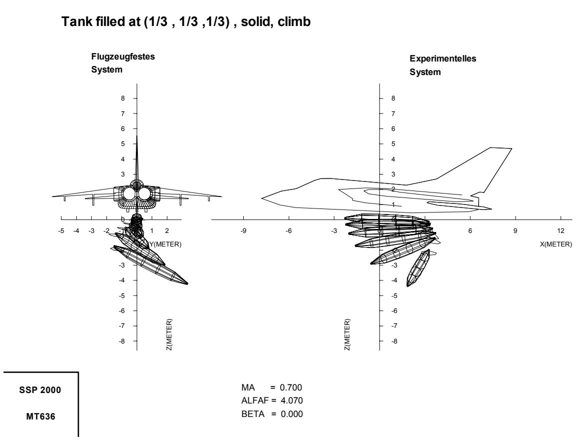


Figure 12: Trajectory without sloshing, partially filled tank, nz = 1g, γ = +45 deg, climb

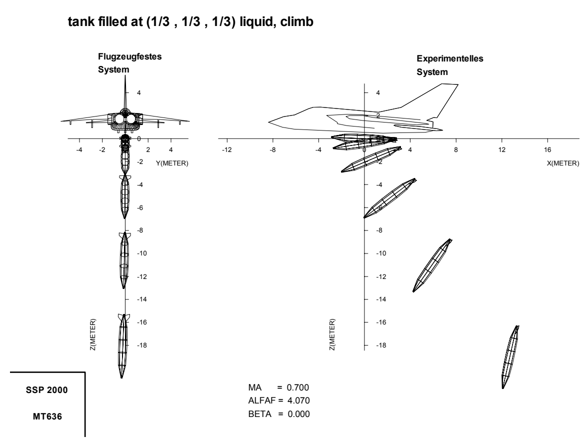


Figure 13: Trajectory with sloshing, partially filled tank, nz = 1g, γ = +45 deg, climb

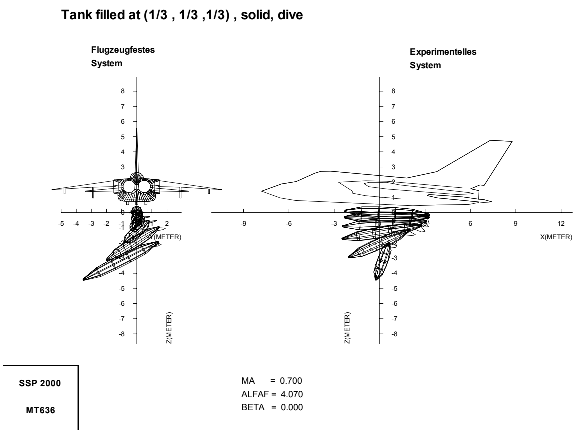


Figure 14: Trajectory without sloshing, partially filled tank, nz = 1g, γ = -45 deg, dive

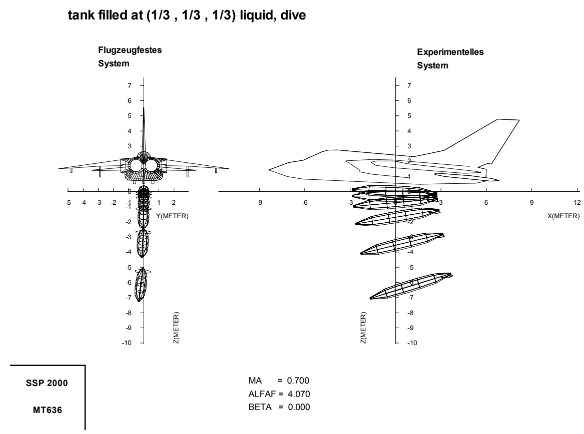


Figure 15: Trajectory with sloshing, partially filled tank, nz = 1g, γ = -45 deg, dive

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DISCUSSION EDITING

Paper No. 18: Prediction of Tank Fuel Sloshing during Jettison

Authors: André Baeten, Rupert Gleissl

Speaker: Andre Baeten

Discussor: G. Akroyd

Question: How do you account for the effect of air in the tank?

Speaker's Reply: Air in the tank leads to a different free surface of the fuel and should be considered as well in the simulation.

Using the particle approach, pressure due to air in the tanks can be implemented by additional pressure loads on each particle close to the free surface. This is a task for further investigations with the particle approach and is envisaged for further improvements of the method.

Discussor: G. Moretti

Question: How much CPU time is required for the trajectory?

Speaker's Reply: The computation of a trajectory using the steady-state slosh subroutine, modified by the particle approach, takes no longer than a few seconds up to one minute.

The CPU time required for a particle approach computation depends on the number of particles involved. For the present example with nearly 10000 particles, one second in real time requires ca. two hours CPU time.

Discussor: A. Cenko

Question: The previous paper on tank sloshing showed the tank pitching up, yours shows it pitching down.

How do you account for the discrepancy?

Speaker's Reply: In the current store-release-program, fuel impact on the tank walls is not yet considered. The trajectories presented here only take into account the center of gravity shift and changing moments of inertia. The particle approach considers impacts on the walls, soon these effects will be implemented in the simulation routine as well. The pitch up due to fuel impact is correct and leads to slightly different trajectories than presented here.

Discussor: F. Ljungberg

Question: Have you analyzed how particle size affects your dynamic results?

Speaker's Reply: Yes, several particle sizes have been used for identical fill levels of a cylinder. The results differ remarkably in dependency of the particle diameter in terms of waves on the surface and separated fuel masses:

For large particles, the dynamics of the liquid is too restricted due to inertia effects as the particle mass increases with the diameter. But for a certain size limit, details like surface waves or sloshing modes are clearly visible an correspond well to liquid dynamics.